ANSWER KEY

(1) D
(2) B
(3) B
(4) B
(5) A
(6) A
(7) C
(8) C
(9) C
(10) A

(11) B = $\begin{bmatrix} \frac{7}{2} & 9 \\ \frac{3}{2} & 4 \end{bmatrix}$ and A = $\begin{bmatrix} 8 & -18 \\ -3 & 7 \end{bmatrix}$. If AB = I_2 = BA then B = A^{-1}.

$$\begin{aligned} AB &= \begin{bmatrix} 8 & -18 \\ -3 & 7 \end{bmatrix} \begin{bmatrix} \frac{7}{2} & 9 \\ \frac{3}{2} & 4 \end{bmatrix} \\ &= \begin{bmatrix} (8 \times \frac{7}{2} + -18 \times \frac{3}{2}) & (8 \times 9 + -18 \times 4) \\ (-3 \times \frac{7}{2} + 7 \times \frac{3}{2}) & (-3 \times 9 + 7 \times 4) \end{bmatrix} \\ &= \begin{bmatrix} (28 + -27) & (72 + -72) \\ (\frac{-21}{2} + \frac{21}{2}) & (-27 + 28) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

The result is the same from matrix product BA.

(12) Given: $C(x) = 0.0001x^3 - 0.06x^2 + 300x + 10,000$
$R(x) = 350x$

(i) Marginal cost function $\equiv C'(x)$

$$\begin{aligned} C'(x) &= \frac{d}{dx}(0.0001x^3) - \frac{d}{dx}(0.06x^2) + \frac{d}{dx}(300x) + \frac{d}{dx}(10,000) \\ &= 0.0001 \times 3x^{3-1} - 0.06 \times 2x^{2-1} + 300 \times 1x^{1-1} + 0 \\ &= 0.0003x^2 - 0.12x + 300 \end{aligned}$$

(ii) Marginal revenue function $\equiv R'(x)$

$$R'(x) = \frac{d}{dx}(350x) = 350 \times 1x^{1-1} = 350$$

(iii) Marginal profit function $\equiv P'(x) = R'(x) - C'(x)$

$$\begin{aligned} P'(x) &= R'(x) - C'(x) = 350 - [0.0003x^2 - 0.12x + 300] \\ &= -0.0003x^2 + 0.12x + 50 \end{aligned}$$

(13) The information provided gives two points along the linear demand curve: $(600, 50)$ and $(800, 40)$, in the form (x = phones, p = price).

Step 1: Define the demand function, $p(x)$, using point-slope form: $p - p_1 = m(x - x_1)$ where (x_1, p_1) is some point and $m =$ slope of the demand curve.

$$m = \frac{\Delta p}{\Delta x} = \frac{p_2 - p_1}{x_2 - x_1} = \frac{40 - 50}{800 - 600} = \frac{-10}{200} = \frac{-1}{20}$$

$$\begin{aligned} p - p_1 = m(x - x_1) &= \frac{-1}{20}(x - 600) \\ p &= \frac{-1}{20}(x - 600) + p_1 = \frac{-1}{20}x - \frac{1}{20}(-600) + 50 \\ \therefore p(x) &= \frac{-1}{20}x + 80 \end{aligned}$$

Step 2: Define the revenue function, $R(x) = p(x) \times x \equiv \text{price} \times \text{quantity}$.

$$\begin{aligned} R(x) = p(x)x &= \left[\frac{-1}{20}x + 80\right]x \\ &= \frac{-1}{20}x^2 + 80x \end{aligned}$$

(14)

(i) Recall, the limit of the difference quotient is defined as: $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$\begin{aligned} \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \underbrace{\frac{(\sqrt{x+h} + \sqrt{x})}{(\sqrt{x+h} + \sqrt{x})}}_{\text{Rationalize}} \\ &= \lim_{h \to 0} \frac{(x+h) - x}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{1}{(\sqrt{x+h} + \sqrt{x})} \\ &= \frac{1}{(\sqrt{x+0} + \sqrt{x})} = \frac{1}{2\sqrt{x}} \end{aligned}$$

$$
\begin{aligned}
f'(x) &= \frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}\left(x^{\frac{1}{2}}\right) \\
&= \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{\frac{-1}{2}} \\
&= \frac{1}{2\sqrt{x}}
\end{aligned}
$$

(ii) A derivative is the instantaneous rate of change of a function, or the slope of a function's tangent line.

GRADING SCALE

Raw Score	Final Score
TBD	