

This exam is closed book. No graphing calculators or cell phones are allowed. No bathroom breaks are permitted while taking the exam. Good luck!

1 Multiple Choice (2 points each)

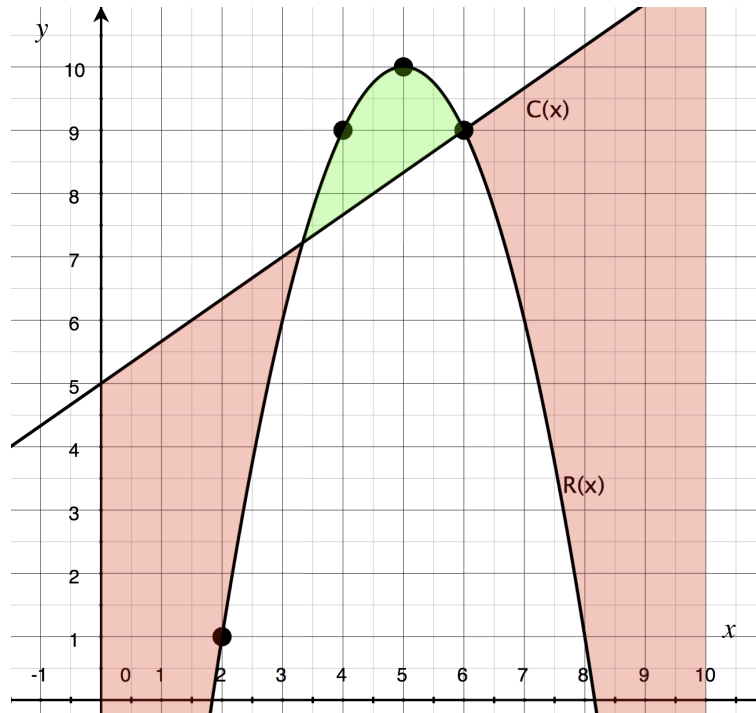
- (1) Find the limit if it exists: $\lim_{x \rightarrow 2} \frac{x^2 + 6x - 16}{x^2 - 2x}$
 (A) indeterminate form (B) 0 (C) 10 (D) 5
- (2) Fill in the blanks: A derivative represents the _____ of _____ on _____.
 (A) marginal effect, dy, dx (C) marginal effect, dx, dy
 (B) average effect, dy, dx (D) average effect, dx, dy
- (3) Suppose that the value V of a car depreciates with time t (measured in months) at the rate $V(t) = 7.5 + e^{-0.10t}$. Find $\lim_{t \rightarrow \infty} V(t)$.
 (A) 0 (B) 5 (C) 7.5 (D) 10

For questions (4)–(8), assume a firm faces the total cost function $C(x) = 5\sqrt{x}$ and total profit function $P(x) = -3x^2 + 9x + 9$.

- (4) Find the average cost between $x = 5$ and $x = 20$ units of output.
 (A) $\frac{1}{4}$ (B) $\frac{\sqrt{5}}{15}$ (C) $\frac{1}{3}$ (D) $\frac{\sqrt{5}}{3}$
- (5) Find the marginal cost at $x = 5$ units of output.
 (A) $\frac{5}{2\sqrt{5}}$ (B) $\frac{1}{2\sqrt{5}}$ (C) $5\sqrt{5}$ (D) $\frac{5}{\sqrt{5}}$
- (6) At what output level is profit maximized?
 (A) $x = \frac{3}{2}$ (B) $x = 1, -1$ (C) $x = \sqrt{3}$ (D) $x = 1$
- (7) Suppose the firm in question increases production, from $x = 2$ to $x = 3$. What is the marginal profit of producing the 3rd unit?
 (A) -\$9 (B) \$0 (C) -\$3 (D) \$3
- (8) Given $C(x)$ and $P(x)$ above, choose a feasible total revenue function $R(x)$:
 (A) $x^2 + 5\sqrt{x}$ (B) $p(x) * x$ (C) $-3x^2 + 9x + 5\sqrt{x}$ (D) $-3x^2 + 9x + 5\sqrt{x} + 9$
- (9) Find the derivative of $g(x) = -8x^4 + \frac{3}{x^3} - \frac{9}{\sqrt{x^5}} + 10x$.
 (A) $g'(x) = -32x^3 - 9x^{-4} + \frac{45}{4}x^{-\frac{9}{4}}$ (B) $g'(x) = -32x^3 - 9x^{-4} + \frac{45}{2}x^{-\frac{7}{2}} + 10$
 (C) $g'(x) = -32x^3 - 9x^{-4} + \frac{18}{5}x^{-\frac{7}{2}}$ (D) $g'(x) = -32x^3 - 9x^{-2} + \frac{45}{2}x^{-3} + 10$
- (10) Find the derivative of $u(x) = \frac{1}{x+1}$.
 (A) $u'(x) = \frac{-1}{x+1}$ (B) $u'(x) = \frac{-1}{x^2+1}$ (C) $u'(x) = \frac{-1}{x^2+2x+1}$ (D) $u'(x) = \frac{1}{(x+1)^2}$

2 Short Answer (3 points each)

Remember: number problems clearly, show all of your work, and circle your final answer.



(11) Refer to the graph above.

- (a) Write an equation describing the *linear* total cost function $C(x)$.
- (b) Write a *quadratic* equation describing the total revenue function $R(x)$.

(HINT: $f(x) = mx + b$ and $f(x) = a(x - h)^2 + k$, where (h, k) represents a parabola's vertex.)

(12) Find the firm's *break-even* points, given the cost and revenue curves above.

(HINT: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.)

(13) Solve the following:

- (a) Find the firm's marginal revenue function, $R'(x)$.
- (b) Show that marginal revenue is zero ($R'(x) = 0$) when revenue is maximized.

(14) Find the firm's profit function, $P(x)$, and then find *what* maximum profit is at the point where $P'(x) = 0$.