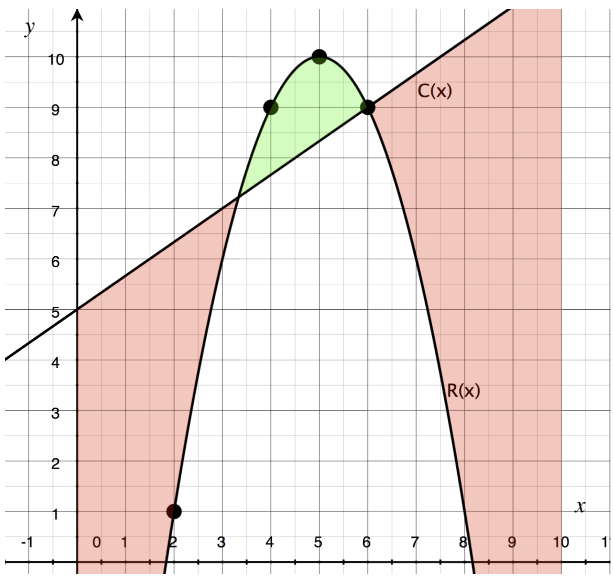
ANSWER KEY

(1) D
(2) C
(3) C
(4) D
(5) A
(6) A
(7) C
(8) D
(9) B
(10) C

(11)



(a) A *linear* function is of the form: $y = mx + b$ where $m =$ the slope of the line and $b =$ the y-intercept of the line.

From the graph one can clearly see that the linear function $C(x)$ has a y-intercept of 5. Thus $b = 5$.

We must now calculate the slope. In order to do so we need a second point (our first point is the y-intercept, $(0, 5)$). From the graph we can clearly see another point along $C(x)$ is $(6, 9)$. Then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-5}{6-0} = \frac{4}{6} = \frac{2}{3}$$

Thus we have $C(x) = mx + b = \frac{2}{3}x + 5$.

(b) A *quadratic* function, in vertex form, is written $f(x) = a(x-h)^2 + k$ where (h, k) are the coordinates of the parabola's vertex. The vertex is the maximum of the curve when $a < 0$ and the minimum when $a > 0$. From the graph of $R(x)$ we can clearly see the parabola is upside down and thus the vertex represents a maximum. The maximum occurs at the vertex whose coordinates are $(5, 10)$. Thus $R(x) = -a(x-5)^2 + 10$.

Assume $a = -1$ so that $R(x) = -(x-5)^2 + 10$. We can test to confirm this is correct by evaluating the function $R(x)$ at some known point—thus we know what it should evaluate to for a given x value. From the graph we can clearly see the point $(4, 9)$ lies on the parabola.

Then,

$$\begin{aligned}
R(x) &= -(x-5)^2 + 10 \\
R(4) &= -(4-5)^2 + 10 = -(-1)^2 + 10 = -1 + 10 \\
R(4) &= 9.
\end{aligned}$$

Therefore our functional form $R(x) = -(x-5)^2 + 10$ is correct.

(12) To find the firm's *break-even* points, we simply set $C(x) = R(x)$ and solve for x. This is where total cost equals total revenue and our firm is literally breaking even.

$$\begin{aligned}
C(x) = 5 + \frac{2}{3}x = R(x) = -(x-5)^2 + 10 \\
0 &= -x^2 + \frac{28}{3}x - 20 \\
0 &= -3x^2 + 28x - 60 \text{ \}multiply both sides by 3 to remove denominator} \\
0 &= (-3x + 10)(x - 6) \text{ \} to solve by factoring} \\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ \} to solve by quadratic formula} \\
x &= \frac{-28 \pm \sqrt{28^2 - 4(-3)(-60)}}{2(-3)} \\
&= \frac{-28 \pm \sqrt{784 - 720}}{-6} = \frac{-28 \pm 8}{-6} \\
x &= \frac{10}{3} \text{ or } 6.
\end{aligned}$$

(13)

(a) To find the firm's marginal revenue function, $R'(x)$, we simply differentiate $R(x)$.

First, expand $R(x) = -(x-5)^2 + 10$:

$$\begin{aligned} R(x) &= -(x^2 - 10x + 25) + 10 \\ &= -x^2 + 10x - 25 + 10 = -x^2 + 10x - 15. \end{aligned}$$

Now take derivative of each term separately:

$$\begin{aligned} R'(x) &= \frac{d}{dx} - x^2 + \frac{d}{dx}10x - \frac{d}{dx}15 \\ &= -2x + 10 - 0 \\ R'(x) &= -2x + 10. \end{aligned}$$

(b) To show that marginal revenue is zero ($R'(x) = 0$) when revenue is maximized, solve for x in the equation $R'(x) = 0 = -2x + 10$:

$$\begin{aligned} R'(x) = 0 &= -2x + 10 \\ 2x &= 10 \\ x &= 5. \end{aligned}$$

Therefore marginal revenue equals zero when $x = 5$ or $R'(5) = 0$. This is the same point at which revenue is maximized. Note that this is the x-coordinate of the vertex!

(14) To find the firm's profit function, $P(x)$, recall that profit is equal to total revenues minus total costs: $P(x) = R(x) - C(x)$.

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= [-x^2 + 10x - 15] - [5 + \frac{2}{3}x] \\ &= -x^2 + 10x - \frac{2}{3}x - 15 - 5 \\ P(x) &= -x^2 + \frac{28}{3}x - 20 \\ P(x) &= -3x^2 + 28x - 60 \; \} \text{ if we choose to multiply by 3 to remove denominator} \end{aligned}$$

To find where maximum profit occurs, we must solve $P'(x) = 0$.

First find $P'(x)$ by either differentiating $P(x)$, or by taking $R'(x) - C'(x)$:

$$\begin{aligned} P'(x) &= \frac{d}{dx} - x^2 + \frac{d}{dx}\frac{28}{3}x - \frac{d}{dx}20 \\ &= -2x + \frac{28}{3}. \end{aligned}$$

or

$$\begin{aligned} P'(x) = R'(x) - C'(x) &= [-2x + 10] - [\frac{2}{3}] \\ &= -2x + \frac{30}{3} - \frac{2}{3} \\ &= -2x + \frac{28}{3}. \end{aligned}$$

Now solve $P'(x) = 0$:

$$\begin{aligned} P'(x) = 0 &= -2x + \frac{28}{3} \\ 2x &= \frac{28}{3} \\ x &= \frac{28}{6} = \frac{14}{3} \end{aligned}$$

Thus $x = \frac{14}{3}$ is *where* maximum profit occurs, or the level of output. But we are asked to find *what* maximum profit is when it's maximized, thus we must evaluate $P(x)$ at this maximum point.

$$\begin{aligned} P(\frac{14}{3}) &= -(\frac{14}{3})^2 + \frac{28}{3}(\frac{14}{3}) - 20 \\ &= \frac{-196}{9} + \frac{392}{9} - 20 \\ &= \frac{16}{9} \end{aligned}$$

GRADING SCALE

Raw Score	Final Score
28	100
27.5	99.5
26	98
21.5	93.5
20.5	92.5
20	92
17.5	89.5
16.5	88.5
15.5	87.5
14	86
13.5	85.5
12.5	84.5
12	84
11	83
10	82
9	81
8.5	80.5
8	80
7.5	79.5
6.5	78.5
6	78
5	77
2	74