

- (1) A
- (2) D
- (3) C
- (4) B
- (5) A
- (6) D
- (7) C
- (8) A
- (9) B
- (10) B

(11)

The graph of  $g(x) = -\sqrt{x+1}-2$  (red line in Figure 1) is composed of three transformations to the elementary function  $f(x) = \sqrt{x}$ , where  $g(x) = af(x-h)+k$  and each variable  $a, h, k$  corresponds to a particular transformation. The starting elementary function  $f(x) = \sqrt{x}$  is in black.

Transformation	Variable and value	Graph plot
Reflection over x-axis	$a = -1$	Black, dotted
Horizontal shift	$h = -1$	Blue, dotted
Vertical shift	$k = -2$	Red

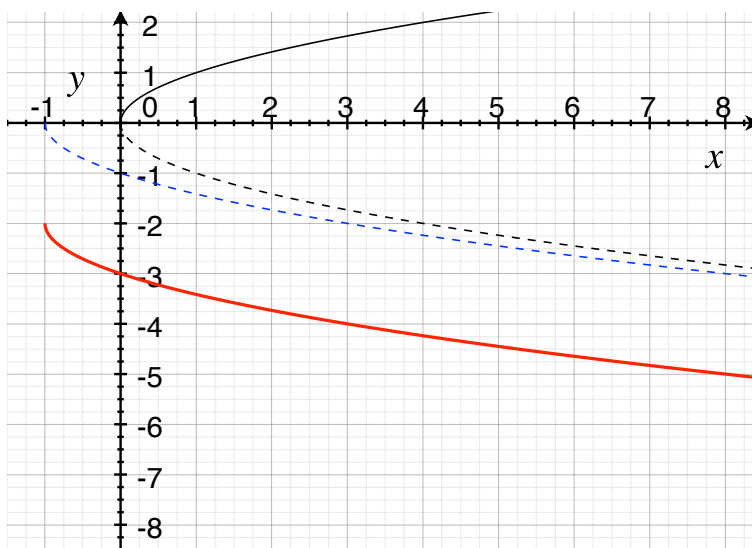


Figure 1: Graph of  $f(x)$  and its transformations into  $g(x)$ .

(12)  $A = P(1 + \frac{r}{m})^{mt}$  where  $A = 2,500$ ,  $P = 2,000$ ,  $r = 0.05$ , and  $m = 4$  (because the

compounding frequency occurs quarterly, i.e. 4 times a year).. Then

$$\begin{aligned}
 A &= P\left(1 + \frac{r}{m}\right)^{mt} \\
 2,500 &= 2,000\left(1 + \frac{0.05}{4}\right)^{4t} \\
 1.25 &= (1.0125)^{4t} \\
 \ln(1.25) &= \ln(1.0125^{4t}) = 4t \cdot \ln(1.0125) \\
 \frac{\ln(1.25)}{\ln(1.0125)} &= 4t \\
 17.9628 &= 4t \\
 t &= 4.4907 \text{ years}
 \end{aligned}$$

(13) If  $\mathbf{A} = \begin{bmatrix} 5 & 7 \\ -3 & 8 \\ 2 & -1 \end{bmatrix}$  and  $\mathbf{B} = \begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 2 & 1 & 1 \end{bmatrix}$ , then only  $\mathbf{AB}$  is conformable and

thus a possible matrix product because  $\begin{matrix} \mathbf{A} & \mathbf{B} \\ 3 \times 2 & 2 \times 4 \end{matrix}$  (i.e. the interior dimensions are equal, or  $2=2$ ). The matrix product  $\begin{matrix} \mathbf{B} & \mathbf{A} \\ 2 \times 4 & 3 \times 2 \end{matrix}$  is not conformable because the interior dimensions are not equal, or  $4 \neq 3t$ , and thus there exist too many elements in each row of  $\mathbf{B}$  and not enough elements in each column of  $\mathbf{A}$ .

Therefore, we have the following matrix product operation:

$$\begin{aligned}
 \mathbf{AB} &= \begin{bmatrix} 5 & 7 \\ -3 & 8 \\ 2 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 2 & 1 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} (5 \cdot 1 + 7 \cdot 0) & (5 \cdot 1 + 7 \cdot 2) & (5 \cdot 0 + 7 \cdot 1) & (5 \cdot 2 + 7 \cdot 1) \\ (-3 \cdot 1 + 8 \cdot 0) & (-3 \cdot 1 + 8 \cdot 2) & (-3 \cdot 0 + 8 \cdot 1) & (-3 \cdot 2 + 8 \cdot 1) \\ (2 \cdot 1 + -1 \cdot 0) & (2 \cdot 1 + -1 \cdot 2) & (2 \cdot 0 + -1 \cdot 1) & (2 \cdot 2 + -1 \cdot 1) \end{bmatrix} \\
 &= \begin{bmatrix} 5 & 19 & 7 & 17 \\ -3 & 13 & 8 & 2 \\ 2 & 0 & -1 & 3 \end{bmatrix}
 \end{aligned}$$

$\mathbf{BA} = \text{Non conformable}$

Recall that the exterior dimensions of  $\begin{matrix} \mathbf{A} & \mathbf{B} \\ 3 \times 2 & 2 \times 4 \end{matrix}$  describe the dimensions of  $\mathbf{AB}$ , which are  $3 \times 4$ .

(14) Let  $x_1$  represent the number of \$5 tickets sold and  $x_2$  represent the number of \$10 tickets sold. Lets create a linear system of equations that meet the constraints for each venue. Let  $C$  represent the maximum capacity of each venue, and  $R$  the revenue minimum before the band can earn ticket sales revenue. In other words,

$$\begin{aligned} x_1 + x_2 &= C \\ 5x_1 + 10x_2 &= R \end{aligned}$$

is the linear system we must solve for each concert hall.

<i>Venue</i>	Roxbury	Orpheum	Barrymore
<i>Capacity = C</i>	1,000	2,000	3,000
<i>Rev. Shr. Min. = R</i>	\$5,000	\$ 10,000	\$ 20,000

Next, lets translate the linear system into a matrix equation so that we know how to solve the matrix equation for our variables,  $x_1$  and  $x_2$ :

$$\begin{aligned} \begin{bmatrix} 1 & 1 \\ 5 & 10 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= \begin{bmatrix} C \\ R \end{bmatrix} \\ \mathbf{A}\mathbf{X} &= \mathbf{B} \\ \mathbf{A}^{-1}\mathbf{A}\mathbf{X} &= \mathbf{A}^{-1}\mathbf{B} \\ \mathbf{X} &= \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \mathbf{A}^{-1}\mathbf{B} \end{aligned}$$

Therefore, so solve for  $x_1$  and  $x_2$  for each venue, we must find  $\mathbf{A}^{-1}$ . Lets use the shortcut where if  $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  then  $\mathbf{A}^{-1} = \frac{1}{\det(\mathbf{A})} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$  where  $\det(\mathbf{A}) = ad - bc$ . Then

$$\det(\mathbf{A}) = 1 \cdot 10 - 1 \cdot 5 = 5$$

And so the inverse is

$$\mathbf{A}^{-1} = \frac{1}{5} \begin{bmatrix} 10 & -1 \\ -5 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -0.2 \\ -1 & 0.2 \end{bmatrix}.$$

To solve for ticket sales for each venue we simply multiply  $\mathbf{A}^{-1}$  by each venue's constraints  $\mathbf{B} = \begin{bmatrix} C \\ R \end{bmatrix}$ .

$$\text{Roxbury: } \mathbf{X} = \mathbf{A}^{-1}\mathbf{B} = \begin{bmatrix} 2 & -0.2 \\ -1 & 0.2 \end{bmatrix} \times \begin{bmatrix} 1,000 \\ 5,000 \end{bmatrix} = \begin{bmatrix} 1,000 \\ 0 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\text{Orpheum: } \mathbf{X} = \mathbf{A}^{-1}\mathbf{B} = \begin{bmatrix} 2 & -0.2 \\ -1 & 0.2 \end{bmatrix} \times \begin{bmatrix} 2,000 \\ 10,000 \end{bmatrix} = \begin{bmatrix} 2,000 \\ 0 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Barrymore:  $\mathbf{X} = \mathbf{A}^{-1}\mathbf{B} = \begin{bmatrix} 2 & -0.2 \\ -1 & 0.2 \end{bmatrix} \times \begin{bmatrix} 3,000 \\ 20,000 \end{bmatrix} = \begin{bmatrix} 2,000 \\ 1,000 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

**GRADING SCALE**

Raw Score	Final Score
27	100
26	98.75
21.5	93.125
21.5	93.125
20.5	91.875
17.5	88.125
16.75	87.1875
15.5	85.625
14.5	84.375
13.5	83.125
12.5	81.875
12	81.25
11	80
10.5	79.375
10	78.75
9.5	78.125
8.5	76.875
7	75
6	73.75