ANSWER KEY

(1) C
(2) D
(3) B
(4) A
(5) B
(6) D
(7) B
(8) A
(9) A
(10) A

(11) If MN = I_3 = NM then N = M^{-1}.

$$\text{MN} = \begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & 0 \\ 0 & -1 & 2 \end{bmatrix} \times \begin{bmatrix} -1 & -1 & 1 \\ -2 & -1 & 2 \\ -1 & -1/2 & 3/2 \end{bmatrix}$$

$$= \begin{bmatrix} (1*-1+-2*-2+2*-1) & (1*-1+-2*-1+2*-1/2) & (1*1+-2*2+2*3/2) \\ (-2*-1+1*-2+0*-1) & (-2*-1+1*-1+0*-1/2) & (-2*1+1*2+0*3/2) \\ (0*-1+-1*-2+2*-1) & (0*-1+-1*-1+2*-1/2) & (0*1+-1*2+2*3/2) \end{bmatrix}$$

$$= \begin{bmatrix} (-1+4-2) & (-1+2-1) & (1-4+3) \\ (2-2+0) & (2-1+0) & (-2+2+0) \\ (0+2-2) & (0+1-1) & (0-2+3) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The result is the same from matrix product NM.

(12) The cost structure for each guitar type is:

Model	*Labor*	*Material*
A	\$30	\$20
B	\$40	\$30

The weekly cost allocations are:

Cost	*Week*1	*Week*2
Labor	\$1,800	\$1,750
Material	\$1,200	\$1,250

Our system of equations and matrix equation become:

$$\begin{aligned} Labor:&\quad 30a + 40b = k_1 \\ Material:&\quad 20a + 30b = k_2 \end{aligned} \Rightarrow \begin{bmatrix} 30 & 40 \\ 20 & 30 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} \Rightarrow \text{AX} = B$$

To solve for a and b, we solve the matrix equation: $\mathrm{X} = \mathrm{A}^{-1}\mathrm{B}$. First, find A^{-1}.

$$\mathrm{A}^{-1} = \det(A)\begin{bmatrix} 30 & -40 \\ -20 & 30 \end{bmatrix} = \frac{1}{30 * 30 - 40 * 20}\begin{bmatrix} 30 & -40 \\ -20 & 30 \end{bmatrix}$$
$$= \frac{1}{100}\begin{bmatrix} 30 & -40 \\ -20 & 30 \end{bmatrix} = \begin{bmatrix} 0.3 & -0.4 \\ -0.2 & 0.3 \end{bmatrix}$$

Week 1 allocations for model A and model B:

$$\mathrm{X} = \mathrm{A}^{-1}\mathrm{B} \Rightarrow \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0.3 & -0.4 \\ -0.2 & 0.3 \end{bmatrix} \times \begin{bmatrix} 1,800 \\ 1,200 \end{bmatrix} = \begin{bmatrix} 60 \\ 0 \end{bmatrix}$$

Week 2 allocations for model A and model B:

$$\mathrm{X} = \mathrm{A}^{-1}\mathrm{B} \Rightarrow \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0.3 & -0.4 \\ -0.2 & 0.3 \end{bmatrix} \times \begin{bmatrix} 1,750 \\ 1,250 \end{bmatrix} = \begin{bmatrix} 25 \\ 25 \end{bmatrix}$$

(13) This question asks you to *approximate* a change in a function. Therefore use the differential equation: $dR = R'(x)dx$.

$$\begin{aligned} dR &= \text{unknown} \\ R'(x) &= \text{marginal revenue at } x \Rightarrow R'(55,700) = \$18,000 \\ dx &= \text{change in units} \Rightarrow dx = 56,300 - 55,700 = 600 \end{aligned}$$

Substitute into differential:

$$dR = R'(x)dx = \$18,000 \times 600 = \$10,800,000$$

(14)

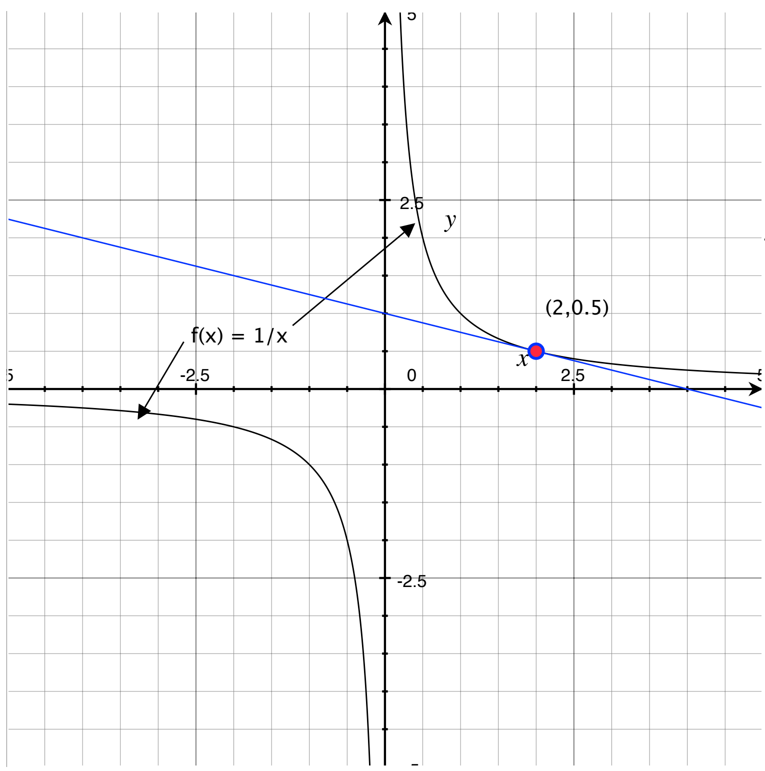
(i) Recall, the limit of the difference quotient is defined as: $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$$\begin{aligned} \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} &= \lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h} \\ &= \left.\lim_{h\to 0}\frac{\frac{1}{x+h}\times\left(\frac{x}{x}\right)-\frac{1}{x}\times\left(\frac{x+h}{x+h}\right)}{h}\right\} \text{Multiply by common denominator factors} \\ &= \lim_{h\to 0}\frac{\frac{x-(x+h)}{(x+h)x}}{h} = \lim_{h\to 0}\frac{\frac{-h}{(x+h)x}}{h} = \lim_{h\to 0}\frac{-1}{(x+h)x} = \frac{-1}{(x+0)x} = \frac{-1}{x^2} \end{aligned}$$

$$\begin{aligned} f'(x) &= \frac{d}{dx}(\frac{1}{x}) = \frac{d}{dx}(x^{-1}) \\ &= -1x^{-1-1} = -x^{-2} \\ &= \frac{-1}{x^2} \end{aligned}$$

(ii) $f'(x) = \frac{-1}{x^2} \Rightarrow f'(2) = \frac{-1}{2^2} = -\frac{1}{4} \equiv$ slope of tangent line at $x = 2$. A point along the tangent line is $(2, f(2))$ where $f(2) = \frac{1}{2}$. Given slope and point, we use the point-slope form to find the equation of the tangent line:

$$\begin{aligned} y - y_1 = m(x - x_1) \Rightarrow y - \frac{1}{2} &= -\frac{1}{4}(x-2) = -\frac{x}{4} + \frac{1}{2} \\ y &= -\frac{x}{4} + \frac{1}{2} + \frac{1}{2} \\ y &= -\frac{x}{4} + 1 \end{aligned}$$



$f(x) = \frac{1}{x}$ Table of values

x	$y = f(x)$
-5	-0.2
-2	-0.5
-0.5	-2
0	vertical asymptote
0.5	2
2	0.5
5	0.2

GRADING SCALE

Raw Score	Final Score
28.5	100
25.5	97.3
24	95.95
21.5	93.7
18.5	91
16	88.75
15	87.85
15	87.85
14	86.95
13.5	86.5
13	86.05
12.5	85.6
12	85.15
11.5	84.7
11	84.25
10.5	83.8
10	83.35
9	82.45
8.5	82
8.5	82
8	81.55
7.5	81.1
7	80.65
5	78.85
4	77.95
3	77.05
2	76.15